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THE INFLUENCE OF STANDARDIZED TESTS ON THE CURRICULUM IN ARITHMETIC[1]

By CLIFFORD B. UPTON
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With the beginnings of experimental education and research in educational psychology a movement started which has had considerable influence on the teaching of arithmetic. Out of this movement grew the standardized tests in arithmetic as we find them to-day. We have had considerable assistance from these tests and more or less harm; it is the harm that comes from them that I wish to discuss in this paper. Unfortunately teachers the country over, being unfamiliar in general with the larger developments in the teaching of arithmetic, have looked upon the standardized test in arithmetic as a kind of authoritative guide as to what should be done in the teaching of this subject. From their viewpoint, the arithmetic they had taught was good if the pupil succeeded in passing a standardized test; otherwise the subject matter and the teaching were poor. Consequently teachers have come to regard many of these tests as outlines of the minimum essentials in number work and therefore guides to the particular topics which should be given most emphasis. This is, however, not the case.

Many of these tests were prepared by those who had little familiarity with the larger problems in the teaching of arithmetic and who had given no particular study to an evaluation of its subject matter before undertaking their work. Certain of the authors of these tests did possess, however, a thorough familiarity with statistical technique as applied to educational problems which they hastened to adapt to arithmetic without carefully selecting in advance the problems to which they applied it. The result is that these standardized tests contain considerable material which is not representative of the best practice in the

[1] An address delivered at the meeting of the National Council of Teachers of Mathematics at Cincinnati on February 21, 1925.

teaching of arithmetic, which tends to standardize topics which should not be taught, and which consequently handicaps certain efforts to reorganize the curriculum in this subject along modern lines.

Recently I asked a teacher of the fifth grade what denominate numbers she was teaching and she replied, "Only those which are found in the ——— Test which our pupils are given twice a year." Thus this test became her authoritative curriculum so far as denominate numbers were concerned, limiting her work to the adding of feet and inches, and years and months. I then asked her what she was doing in mensuration, such as finding areas, and she replied, "Very little, because the tests don't include such work."

Before entering upon the details of this paper I wish to say that the testing movement has made a definite contribution to the teaching of arithmetic in supplying ideas about standards of attainment and the means of measuring the progress of pupils which had not previously been available. Had it been possible from the beginning for test makers to have worked together with those who had specialized in the teaching of arithmetic, many of the mistakes would have been avoided that must now be corrected. But all new movements develop in this uneconomical way, mistakes being noted and corrected only after extended experience. Having passed through the period of adolescence we may now reasonably expect the testing movement in arithmetic to give us much more sane and efficient service in the near future.

In addition to the various standardized tests in arithmetic, I shall include in this discussion the better known practice exercises in this subject as well as the more important intelligence and achievement tests which have incorporated problems in arithmetic.

Let us first take the topic of addition as we find it in these various tests. There seems to be no uniformity whatever as to the degree of difficulty which an addition problem should possess. In one of these tests the goal seems to be the ability to add 9 numbers of 3 orders each as shown in Example 1. In another test the goal is increased to 16 addends as in Example 2. One of the practice exercises gives 13 addends of 6 orders each as in Example 3, while another gives 14 addends of 3 orders each as shown in Example 4.

Numerous variations of the first four examples are found in other tests. At the other extreme one of the achievement tests considers Example 5 the final goal in addition. Evidently the authors of this particular test like them fat rather than lean. It goes without saying that the standards represented by these five examples cannot all be right.

1.	2.	3.	4.	5.
927	.49	226315	916	791529572
379	.28	841682	699	865280183
756	.63	657159	974	437643359
837	.95	840445	473	
924	1.69	571145	134	
110	.22	642735	855	
854	.33	178068	581	
965	.36	376370	768	
344	1.01	911917	586	
	.56	745523	893	
	.88	189933	135	
	.75	458671	624	
	.56	310367	328	
	1.10		535	
	.18			
	.56			

In order to relieve eyestrain it is customary in business practice to use commas to separate large numbers into groups of three figures each, as in this number: 226,315. It should be noted in Examples 3 and 5 that this practice has not been followed, thus giving the eye no resting point whatever. It would have been preferable in Example 3 to have had the numbers represent dollars and cents, thus bringing this exercise more in harmony with conditions outside the school room.

The question of proper standards is still further complicated by the fact that certain practice exercises are giving a great amount of emphasis to problems which involve considerable irregularity of outline such as Example 6. In such extremes as

6.	7.
3738	$37.38
634	6.34
59399	593.99
964	9.64
42	.42
825585	8255.85
407	4.07
52	.52
513	5.13

this we must remember that we are subjecting pupils to a great amount of nerve and eyestrain to keep the numbers in their proper columns, a strain which we do not impose even upon

an expert bookkeeper who always writes such numbers in account books which are especially ruled so that the figures easily follow in straight columns. If an exercise like Example 6 is included in a test it would be far more sensible to write it as shown in Example 7 since the addition of numbers of various orders occurs most often in accounting or in banking, where the addends are dollars and cents.

When it comes to the addition of decimals the situation is still more complicated. We find the following as examples of this complication:

8. 33 + .0033 + 330000 + 33.33

9. 4.833 + 17 1/2 + 3 2/5 + 21.9

10. .14 + .8 4/5 + .5 7/8

11. 0.14 + 0.8 4/5 + 0.5 7/8

I know of no situations in practical life which justify such extremes as Example 8 and still less a problem like Example 9. If by any possibility a problem like Example 9 actually existed it would be the result of measurements of some sort but it is very difficult to conceive of any form of measuring which would be constantly mixing common and decimal fractions. While Example 9 is bad, Example 10 is infinitely worse, and is certainly a product of the imagination rather than a problem from any actual life situation. Yet the teacher whose pupils have once been given a test which incorporates Examples 8 and 9, or the test which gives Example 10, will ever after drill upon that sort of material to make certain that her pupils will make a good record.

It should be noted that if Example 10 were a real problem it would represent the sum of three measurements of some kind and would be written by a careful computer as shown in Example 11, a zero being placed before each decimal point to make certain that the decimal point would not be overlooked. The importance of such a practice is readily appreciated by an illustration from the field of medicine. A physician, prescribing a powerful medicine in terms of metric units, as is often done, would always indicate the dose by writing 0.25 grams instead of .25 grams, the zero serving to call attention to the decimal point in case it is written indistinctly. In a situation like this it would

be very serious to give 25 grams of a certain medicine when only 0.25 grams was intended. Example 10 is, therefore, not only a fictitious problem but it is written carelessly as well.

Examples 12 and 13, which are taken from two prominent tests, are still more puzzling since they represent a form of writing the addition of decimals which is never found in practice.

12.	13.	14.
113.46	37.846	113.4600
49.6097	1.02	49.6097
19.9	8.109	19.9000
9.87	70.61	9.8700
.0086		.0086
18.253		18.2530
6.04		6.0400

It is evident that Example 12 does not represent the addition of dollars and cents. The same may be said of Example 13, since in all ordinary commercial addition, such as the making out of bills, the fractions of a cent are not accumulated as would be indicated if the items in Example 13 were money. In such cases a fraction of a cent equal to one-half or more is always counted as another cent while fractions less than one-half are dropped.

The only situations where one finds calculations involving money, carried to more than two decimal places, are in highly technical computations such as are found in cost accounting, in the calculation of tax rates, or in quotations on foreign money, in which cases *all* items in the same problem are carried out to the *same* number of decimal places. If Example 12 represented such a situation it would then be written as shown in Example 14. I should add, however, that the addition of money items carried out to several decimal places is a far less frequent operation than the multiplication or division of such items, and in offices where such extensive calculations with decimals occur, the calculating machine is usually employed.

Examples 12 and 13 are equally objectionable if we consider them as representing measurements of length, weight, area, or volume, instead of money. Any of these is an extreme possibility, since our units of measure are not divided decimally, and since the metric system is not generally used in this country. The modern surveyor, however, sometimes uses a steel tape in which the foot is divided decimally, while in the physical laboratory or in the U. S. Coast and Geodetic Survey we often find

measurements carried out to four or more decimal places, but in all these situations the important thing to remember is that in a given problem all measurements are made with the same degree of accuracy and, consequently, are recorded with the same number of decimal places. Hence, if Example 12 represented such measurements, it would be written as shown in Example 14. If a scientist in measuring two copper plates correct to thousandths of an inch found the thickness of one to be exactly 0.4 inches and that of the other to be 0.372 inches he would record his first measurement as 0.400 to make clear that in his measurement he had not ignored the second and third decimal places. It is just as important for him to record *each* of his measurements to three decimal places as it is for us always to use two decimal places in writing cents. We could write \$7.60 as \$7.6, but to save all question about the second decimal place we always write it as \$7.60. Examples 12 and 13, therefore, are not representative of real conditions either in computing or in measuring.

The most likely situation that occurs to me which would involve the addition of decimals of various orders is that which would arise if one should add several mixed numbers such as $4\frac{1}{4} + 6\frac{1}{2} + 3\frac{1}{8}$, by reducing them to decimals, as shown in

15.	16.
4.25	4.250
6.5	6.500
3.125	3.125

Example 15, but even in such a case a careful computer would not omit the zeros but would write them as in Example 16.

At this point I wish to stop a moment to speak of the subject of checking so far as the operation of addition is concerned. The practical way to check addition is to add each column in the opposite direction. This is the way addition is checked in life by all practical computers. Yet one of the prominent practice exercises tells the pupil to check his addition as follows:

\$1.10	First find the sum		Add part of the
\$2.85	of the first 4 num-		numbers and write
\$3.75	bers which is	\$14.12	the sum.
\$6.42			
\$1.49	Then find the sum		Add the rest of the
\$2.25	of the rest of the		numbers and write
\$7.50	numbers which is	\$11.24	the sum.
	The sum of all the		Then add these 2
\$25.36	numbers is	\$25.36	sums.

Such a practice is indefensible. It should also be noted in this particular example that the $ sign is repeated with each addend, a practice found nowhere in life.

It is of special interest to note that none of the tests tell the pupil to check at all. In fact, they all give time standards which make it impossible for the child to check and finish the work within the time assigned. In other words, every standardized test in arithmetic to-day encourages the practice of not checking problems in addition by forcing standards upon us which make checking impossible. This is quite contrary to life because in all modern business practice, even where adding machines are used, checking is constantly in evidence. The world is not absolutely sure of its results after a single working of a problem. Business men would soon be ruined if bids, estimates, and contracts were not most carefully checked. No one but an educator, who knows little about modern business practices in arithmetic, would venture to depend upon his results after a single working of a problem. It is certainly high time for us seriously to consider whether all time standards in modern tests should not be revised to make allowance for checking.

Passing to subtraction we find a similar wide variation in standards of difficulty. One achievement test is quite satisfied with Example 17 as the maximum accomplishment. A prominent intelligence test suggests Example 18, while two of the standardized arithmetic tests contribute Examples 19 and 20.

17.	18.	19.	20.
971	37344	107795491	80836465
536	14853	77197029	49178036

Certainly Example 17 is too simple for pupils beyond the third grade and undoubtedly Examples 19 and 20 are too extreme for pupils anywhere in the elementary school. It should be observed that the comma used to separate a large number into periods has been omitted in Examples 18, 19 and 20, an omission similarly noted in certain addition problems above.

21.

5704.3⅝ — 695.71

When we come to the subtraction of decimals the situation is still worse. One of the arithmetic tests asks the pupil to subtract Example 21, which is purely an imaginary problem, since

in practice decimal and common fractions are not combined as here indicated. One of the intelligence tests, which has a very reasonable standard in addition, jumps to Examples 22 and 23 for subtraction.

22.	23.
9.2 — 3.00061	9.4 — 4.00083

These particular problems are artificial for the same reasons given above in discussing the examples on the addition of decimals. If the subtrahend is carried to five decimal places, as it would be only in most delicate scientific measurements, then the minuend should be carried out to the same number of places. A form of subtraction somewhat similar to that shown in Examples 22 and 23 occurs in finding the cologarithm of a number, but even there the zeros are understood in the minuend and would be inserted by a careful computer in the case of written work, though cologarithms are usually found mentally.

From my knowledge of computation as found in life I believe that for the great majority of people subtraction involving three decimal places will be quite sufficient. How is any teacher to keep her balance when presumably authoritative tests give problems like the above!

No test makes any provision for checking subtraction and does not allow time for it even if it were desired.

Turning to multiplication there is like variation in the problems. One standardized test gives Example 24 as its notion of the

24.	25.	26.	27
8246	9402875	904753	3037
29	842	3679	3

maximum difficulty in multiplication, while certain practice exercises increase the limits to Examples 25 and 26. At the other extreme we find a prominent intelligence test quite satisfied with a single multiplier as in Example 27. If the reader will try to find a problem in real life which requires the multiplication of numbers such as are found in Examples 25 and 26 he will search many hours before he finds one that is really satisfactory, if he finds it at all. Such problems make us feel that the authors are advocates of the theory of formal discipline in its most extreme form.

When it comes to the multiplication of decimals we find many puzzles for the conscientious teacher who is seeking to improve her work. One test writes its multiplication problem as shown in Example 28, taking particular care to have the decimal points

28.

$$\begin{array}{r} 53.81 \\ \underline{2.1} \end{array}$$

come one under the other. What havoc it would raise in the school room if all multiplication were done in this way. It would be of interest to know just how children place the partial products when they solve this particular problem. Another test built by a statistician expects the pupil to multiply as in Example 29. It would be extremely interesting to know

29.

$$\begin{array}{r} .0963\frac{1}{3} \\ \underline{.084} \end{array}$$

if this statistician ever in his life met a practical problem which involved anything like the computations represented in Example 29, and if he did, if he got the right answer the first time he worked it. What a waste of time to expect us to teach problems like this!

When we consider the multiplication of mixed numbers we find that three prominent tests have problems like Example 30,

30.

$$\begin{array}{r} 358\frac{1}{4} \\ \underline{26} \end{array}$$

where the multiplicand is the mixed number and the multiplier a whole number. Nowhere in these particular tests does the more frequent problem appear where the multiplier is a mixed number. If we were asked to find the cost of 6 packages of crackers at 12½ cents each we would have a typical life problem where the multiplicand is a mixed number but this situation arises much less often than problems like the following where the multiplier is a mixed number: Find the cost of 5¾ pounds of chicken at 48 cents a pound, or the cost of 2½ yards of ribbon at 30 cents a yard. When standardized tests reverse actual conditions in this way how can we expect teachers, who do not understand these matters, to know what relative emphasis should be given these two topics?

Turning to one of the tests which does give a mixed number as a multiplier we find the extreme case illustrated in Example 31, which is, of course, of theoretical rather than of practical

31.

$$6.48 \times .26\tfrac{3}{8}$$

interest. Further along in the same test we find an exercise like Example 32 which is equally extreme, representing an almost impossible combination of decimal and common fractions.

32.

$$.07\tfrac{3}{8} \times 16\tfrac{1}{4}$$

This same test, apparently to destroy all efforts at good teaching, then gives the problems shown in Example 33, in each of which the multiplier is made a concrete number. What teacher is there who hasn't worked for hours making clear to the child that

33.

$2\tfrac{1}{2}$ in. $\times$ 24 = how many ft.?

$12\tfrac{1}{2}$c $\times$ 1600 = ?

$25\tfrac{3}{4}$c $\times$ 1500 = ?

the multiplier must always be an abstract number, only to have a test like this come along and ruin it all.

When it comes to the checking of multiplication we find one of the practice exercises advocating the method here shown.

34	34
9	34
306	34
	34
	34
	34
	34
306 is right,	34
for 9 34's	34
added =	306

Such a method is of course wholly impractical with a multiplier like 9 and is never used in life. As in the problems in addition and subtraction, none of the standard tests provide time for the checking of problems in multiplication.

Turning to division the great majority of the arithmetic and achievement tests print the problems in short division so that the quotient shall be written above the dividend. Anyone familiar with practical computation knows that it is extremely

inconvenient to write the quotient above the dividend in short division. The only possible excuse anyone has for doing it is to harmonize the school-room practice in short division with that in long division, but in life we find that there are other far more important considerations that prompt us to write the quotient below the dividend rather than above it, and we are never worried or inconvenienced there by the fact that the quotient is written *above* in long division and *below* in short division. Let us review for a moment those situations in life where we have to use short divison. There are three such prominent situations.

(1) The first use of short division is that made in finding the fractional part of a number such as finding ¾ of $2.76. If we work this problem by multiplying first by 3 and then dividing by 4 the easiest way to do it is as shown in Example 34. On the

34.	**35.**
2.76	4/2.76
3	.69
4/8.28	3
2.07	2.07

other hand, if we first divide by 4, and then multiply by 3, the work is most conveniently done as in Example 35. In both cases we have actually saved time by writing the quotient *below* the dividend. It is of interest that on several occasions I have asked groups of teachers to show me how they work this particular problem. Invariably they write the quotient below as has been done here, though in their teaching they put the quotient above.

(2) The second prominent use of short division is in finding averages as shown in Example 36. Again we find it more con-

36.
87
43
26
3/156
52

venient to write the quotient below the dividend. If we do not write the quotient below when we divide by 3, then we must rewrite the sum, 156, before dividing.

(3) The third prominent use of short division is in finding discounts as in this problem: If the list price is \$4.80 and a discount of 25% is allowed what is the net price? The shortest solution of this problem is that given in Example 37.

37.

4/4.80
1.20
3.60

It is thus seen that in each of the above cases time is saved by writing the quotient below the dividend in short division. We had made some progress in this country in getting teachers and pupils to write the quotient below until the standardized tests commenced to be circulated widely. One of the most prominent authorities on arithmetic in this country told me recently that he was finding more and more of a tendency on the part of teachers to write the quotient above the dividend. I attribute this tendency wholly to the influence of the various standardized tests in arithmetic, in this case an extremely unfortunate influence.

Let us now consider the subject of long division in which there is no agreement whatever in the various tests as to the size of the dividend and divisor. In one standardized test Example 38

38.	**39.**	**40.**	**41.**	**42.**
68)43520	4927)1896895	358)291412	17)359	48)1536

is representative of the maximum difficulty of the exercises in division, while the practice exercises present problems as complicated as those illustrated by Examples 39 and 40. At the other extreme we find certain achievement tests satisfied with exercises as simple as those shown in Examples 41 and 42. Certainly there must be a happy medium.

Fortunately the various tests do not give such extreme exercises in the division of decimals as were found in multiplication. Examples 43 and 44 are taken, however, from two of the tests

43. \$12 ÷ 6¼c **44.** 27.58 ÷ ⅞

and represent computations that probably occur nowhere in practical life.

When it comes to the checking of long division we find one of the practice exercises advocating the following method:

$$\begin{array}{r} 20005 \\ 15\overline{)300075} \end{array}$$

Check by multiplying as follows:

$$\begin{array}{r} 15 \\ 20005 \\ \hline 75 \\ 30 \\ \hline 300075 \end{array}$$

Just how the pupil is to get the proper position for the partial products is not explained, and in all probability in practice this skill is never acquired. It goes without saying that such extremes as this should have no place in any course of study.

In examining the exercises in common fractions in the various tests we find such specimens as are shown in Examples 45, 46, 47 and 48, in which fractions with denominators like 7, 14, 5, 9, and 18 are given preference to those with the much more commonly used denominators such as 2, 4, and 8. If one should

45. $1/7 + 1/14 + 3/14$

46. $6/7 - 4/5$

47. $3/6 + 4/12$

48. $17/18 + 2/3 + 5/9 + 1/6 + 1/2$

compare these problems with the 2000 cases of fractions given by Professor G. M. Wilson in his book on *Social and Business Usage in Arithmetic* one's conclusion would be that such situations as are suggested by these four examples probably do not arise practically once in a million times. It should also be noted in these four examples and in Examples 49 and 50, given below,

49.
$1/8 + 1/4 + 1/2$

50.
$3/8 + 5/8 + 7/8 + 1/8$

that considerable prominence is given in all the tests to the writing of common fractions in the horizontal form, whereas the most frequent life need is to add fractions in the vertical form. We must remember that most of the fractions we have to add occur in mixed numbers, as shown in Example 51, and

51.

$$\begin{array}{r} 3\frac{1}{4} \\ 2\frac{5}{8} \\ \hline \end{array}$$

that these are most conveniently added when placed one under the other. When the standardized tests give so much prominence to the horizontal writing of fractions it forces the teacher to give considerable drill on the adding of fractions in this form,

thus reducing the attention needed for the vertical form. Only recently a group of teachers told me that they were forced to spend a great deal of time on the horizontal form in order that their pupils would be successful in getting a good rating in these tests.

The textbooks in arithmetic now used in this country have already done much to simplify the teaching of common fractions and the good example they have set would be far more effective than it is now if the testing movement would only come to its senses in these matters. It is really surprising how little the great majority of people need to know of common fractions for the practical affairs of life. If any prominent city in this country would adopt a course of study in which the subject of common fractions was reduced to its barest essentials and would faithfully follow it for a period of years we would find that such an experiment would have very wide influence. Gradually other courses of study would accept a similar standard and before long we would have the millenium so far as the teaching of common fractions is concerned. And with that would come the greatest relief of all, for many of the arithmetic tests now in use would die, as they should, from inanition.

Among a few of the miscellaneous peculiarities found in these tests is a problem such as is shown in Example 52. Undoubtedly

52.
Find 6¼ % of 80

this problem was given to test the child's knowledge of the fractional equivalents of certain per cents, expecting him to remember that 6¼% of 80 is the same as $\frac{1}{16}$ of 80, or 5. While it is quite worth while to have pupils know that 16⅔% of anything is the same as $\frac{1}{6}$ of it and that 12½% is the same as $\frac{1}{8}$, I doubt extremely the value of forcing children to remember that 6¼% is the same as $\frac{1}{16}$, yet the particular test in question gives only this exceptional form.

In the field of denominate numbers one test gives the oddity illustrated in Example 53. I can think of no life situation where

53.
1⅛ yd. + 1.2 ft. = ? inches

we would measure yards in terms of common fractions and feet in terms of decimal fractions. If we had the foot divided into

tenths, as a surveyor sometimes does, then we never would have any occasion to add $1\frac{1}{8}$ yards to 1.2 feet because the rest of the measurement would also be expressed decimally. In a drygoods store where the yard is measured in eighths, one would never find a foot marked in tenths. Such a problem is purely fictitious.

I should call attention to the fact that the tests which I have reviewed in this article have included only those which deal with the abstract operations with whole numbers, fractions, and decimals, because the most prominent tests in arithmetic limit their exercises to this field, some of them confining the work to whole numbers alone. I have not touched upon the so-called reasoning or problem tests which treat of the applied problems of arithmetic, since these tests are not widely used and such of them as we have are not satisfactory.[1] This means that practically all the formal measurement of arithmetic now done by means of standardized tests is confined to the abstract operations. Little or no attention is given to a corresponding testing of the pupil's ability to solve problems. The effect of this on the present teaching of arithmetic is apparent. Abstract work is greatly emphasized in the schools where standardized tests are given while work in problem solving is more or less neglected. Hence we find in many school rooms to-day a lack of a proper balance between abstract and concrete work that is to a great extent the result of a one-sided development of the testing movement in arithmetic.

In conclusion let me say that the illustrations which have been given in this article are selected from the best known and most widely used tests and scales on the market to-day, the total circulation of which reaches many million copies per year. When I say that these measuring devices are a great influence upon our curriculum in arithmetic I am not speaking lightly. Whether the unusual topics found in many of these tests actually appear in the printed courses of study of our many towns and cities is of relatively little moment because these topics *are* taught whether they appear in the curricula or not, and the reason they are taught is because teachers know that they and their schools are judged in no small measure by the rating their pupils obtain

[1] For a discussion of the difficulties in making a satisfactory reasoning or problem test in arithmetic see the author's article on "Standardized Tests in Mathematics" in the Report of the National Committee on Mathematical Requirements, Chapter XIII, pp. 307-316.

on examinations of this sort. What a pity this is when we really have no test in arithmetic to-day which is a sane and adequate measure of good teaching in this subject. Such tests as we have are greatly influencing our teachers to give an undue emphasis to abstract work and they are also forcing into our teaching many fictitious and extreme problems. A number of topics which it would be desirable to test have not been included at all. Apparently many of these tests were prepared with little appreciation of the kind of arithmetic that is actually used in this busy world of ours.

In seeking to find the philosophy of education underlying many of these tests one is reminded of the statement of that keen observer, Mr. Dooley, who said, "It doesn't make much diff'rence what ye teach a boy so long as he doesn't like it." Had the battery of modern arithmetic tests come under Mr. Dooley's observation I am sure he would have remarked, "It doesn't make much diff'rence what ye put in a test so long as it is somethin' that no wan iver uses." When the time comes that most of the present tests are replaced by others which contain problems that some one *does* use in the quantitative affairs of life, we will then have a great force, far more effective than most of us now realize, to assist in simplifying the curriculum in the oldest of all the school subjects, arithmetic.

INDIVIDUAL INSTRUCTION IN NINTH YEAR ALGEBRA[1]

By C. N. STOKES
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A class of eighteen pupils whose average I. Q. was 93, completes the second semester of algebra without a single failure! On a general test at the end of the course the class ranked fifth among nine fast, one normal, and two slow classes. There were no home assignments. The semester's work was completed by one pupil in ten weeks, the others requiring from ten to eighteen weeks. I want to tell you how it happened.

At the beginning of the second semester of the year 1923-24, a class of slow moving pupils were given to me to be taught the second half of ninth year algebra. The personnel of the class comprised pupils whose ability to do algebra seemed to be a negative quantity. With the exception of three cases, the class was made up of pupils who had failed the course one or two times. It was a critical situation for without a doubt formal algebra had become irksome and it was lack of interest and initiative that held them back. The only wise thing to do was to try something new.

Having heard of the many good qualities of individual instruction, the "Winnetka Plan" with a few changes which were necessary because of local conditions, was decided upon.

The results of this first experiment were so gratifying that it is being continued through the present year, not only with slow moving pupils but also with a class of normal pupils.

The Problem

Since the primary aims of instruction in mathematics are to teach the coming generation to think, to think intelligently and independently and to teach mathemaics that will be of both cultural and practical value, it is our business to find a method of presenting the subject whereby that inevitable fifteen to twenty or even thirty per cent of failures will be eliminated. And perhaps it is not the method that is entirely at fault. It

[1] Read before the National Council of Teachers of Mathematics, Cincinnati, February 21, 1925.

may be the content or the arrangement of the content in our textbooks. Quoting from Rugg and Clark, "What we teach and how we teach it," should command our attention. But does not the whole thing resolve itself into the problem of handling individual differences? How many of us would not maintain that, should we be given the opportunity to teach a homogenous group we could do it with a large degree of efficiency by disregarding almost entirely any particular method or content. With that type of a class we believe we could create interest, initiative, and self-reliance, granting that the lack of these characteristics produce the greatest toll of mortality. Since it is not within human power to select such a group our best efforts must be spent in attempting to acquire the desired results with what we have.

Individual instruction is purported to take care of just such situations as teaching algebra to non-homogeneous groups. And as said before, they do exist for the members of any group approach equality in neither ability nor achievement. They vary from one another by differences which may be small between any two but which will be great between extreme cases. And with these differences come the increased attention to varied capacities, interests, future activities, and an increased variation in the needs of society, which have made a change necessary. We must satisfy the belief that a public education of all children is a public duty. Then since the aims and purposes of individual instruction are to adapt materials and methods of instructon to meet the needs of individuals, the results will be that failures will be reduced, the number of graduates will be increased, discipline will be improved, pupils will be stimulated to work to capacity, initiative and interest will be increased, and unusual ability will be discovered and developed.

The general principles of organization and means of administering individual instruction are as follows:

A. *Promotion.* Each pupil is promoted in each part of the work whenever he completes that part of the work of the subject.

B. *Principles.*

1. *Goals.* The work is divided into small definite units called goals. These goals are known to the pupil as well as to the teacher and must be attainable by the slowest normal diligent pupil. They are subdivided merely for the purpose of measur-

ing progress and are arranged in terms of facts to be known, habits to be acquired, and skills to be developed.

2. *Practice Materials.* There are within each goal practice materials corresponding to each weakness likely to be encountered. They are given in lists of twenty-five and are so arranged in cycles of four that any error or weakness may be definitely located.

All practice exercises are self corrective and the answers are arranged in the order in which the pupil is to solve them.

3. *Tests.* The tests are complete and diagnostic, showing whether or not the pupil has attained all the goals and showing his specific weakness.

Proceedure

The entire course is divided into six main divisions and these are subdivided into goals according to the content of the material. The practice materials for a goal consists of lists of twenty-five exercises with sufficient explanatory material preceding them to prepare the pupil adequately for solving the exercises. This explanatory material has two main features, the complete solution and explanation of several exercises typical of those found in the list to be solved, and the partial solution and directions for completion of several more exercises. This latter feature is self corrective and the pupil in making his corrections is directed to turn to the answer sheet to see if his completion of the partial solution checks. If any mistakes have been made he consults with the instructor who attempts to correct the weakness by bringing in relative material.

The pupil is now ready to start on the list of twenty-five practice exercises. Out of that list he attempts to solve a set consisting of every fourth one, namely, 1-5-9-13-17-21-25. Again he turns to the answer sheets to correct his work. If there is a single mistake he must select another set such as the one beginning with number 2 or 3, and proceed as before. When he solves such a set with 100 per cent efficiency he is ready to take up the work on the next goal. And so on, until he has attained all the goals of one of the main divisions of the course. He then takes a preliminary test which is self corrective, the test covering all the goals of the division that he has completed. He must pass this test with 100 per cent efficiency. Should he fail to do it

he must go back, find the type of exercise wherein he is weak and attain that goal again in the same manner as before. Then he takes a second preliminary test. Should the same error occur again, supplementary practice material is given him, and so on until the preliminary test is passed with complete mastery. After the preliminary test comes the final test which is simply another form of the preliminary test, coverng like it, all goals of the division that has been completed. The instructor gives this test and corrects the work. A mark of 100 per cent is required. If it is not made another trial is given. Should the second one be failed supplementary practice material is given relative to the weakness shown. When a final test is passed the pupil is ready to start on the work of the next main division of the course.

Elements in Technique

The adage that "Anything worth learning at all is worth learning well enough to remember or reproduce the essence of it," surely has its bearing in the teaching of algebra. To state it more specifically, *mastery* is a fundamental element in the teaching of algebra. You noted from the preceding paragraphs that individual instruction, through the methods of administering it, requires complete mastery. It is possible because the pupil can proceed at his own rate and is not permitted to take up new work until he has mastered the task at hand.

Pupils never feel the sense of *failure*. One of the fundamental principles of the idea of individual instruction is based on success psychology. The arrangement of the definite units of work to be accomplished one at a time before proceeding to the next gives the pupil a feeling, with the mastery of a unit, that he is not failing even though he may be progressing slowly. That old phrase, "Well, I'm failing anyhow, what's the use," is forced into the discard. Success in accomplishing the separate definite units of work produce interest, initiative, and a desire to forge ahead. The pupil has the opportunity of seeing his specific ability and consequently has the inclination to work to capacity.

For the fast pupil there is no feeling of just "*marking time.*" This has been the cause of many pupils doing only mediocre work and being passed on when they really had exceptional ability. Nature has endowed human beings with a spirit of trying to ex-

eel and thus having the opportunity to proceed as fast as one can, the unusual pupil is discovered and his ability developed.

Group Discussion is a particular activity on which particular emphasis is placed, but at no time is a topic discussed before the entire class. That would be useless since at no time after the first day of school are all the pupils working on the same thing. But the pupils are permitted and encouraged to work together whenever they find other pupils working on the same thing. The instructor does not take a leading part in the group discussions. His duty is merely to guide so that the group may be able to clear up its own difficulty.

There are no home *assignments* made. The pupil may take the work home and proceed as fast as he can but it is not urged for it is found that there is ample time in the class room period to do all the work necessary to proceed at a normal rate.

What Is Taught

Having in mind the contemporary life of the American boys and girls and the fact that the selection of the material of a ninth year course in algebra must meet existing social needs and be psychologically well founded, the material was prepared to satisfy these criteria as far as possible. Great effort was made to produce a logical sequence and at the same time take care of the required amount of repetition. As far as is possible the problems are real and actual, and non-essentials are omitted. Particular emphasis is placed upon relationships and representations, for a course in secondary mathematics which does not stress the ability to recognize verbally stated relationships between magnitudes, to represent such relationships economically by means of symbols, and to determine such relationships, is not going to prepare the pupil to do constructive work in analyzations, comparisons, generalizations, discriminations and the like.

Content

Division I

A. Measurements.

Goal 1. Tools. The English and Metric systems.
Goal 2. Accuracy in measurements.
Goal 3. Straight lines and line-segments.
Goal 4. Use of cross-section paper.
Goal 5. Addition of line-segments. Representations.

B. Formulas.

Goal 6. Areas.
Goal 7. Evaluations.
Goal 8. Algebra vs. arithmetic. Short-hand rule.
Goal 9. Use in business.
Goal 10. Dependence.
Goal 11. Interpretation of verbal descriptions.
Goal 12. Interpretation of equations. Types.
Goal 13. Use of powers and roots.

Division II

A. Graphing.

Goal 1. The table.
Goal 2. The bar graph
Goal 3. The straight-line graph.
Goal 4. The curve-line graph.
Goal 5. Graphing relationships.
a. The formula. Dependent and independent variables.

B. Signed numbers.

Goal 6. Opposite magnitudes. Positive and negative numbers.
Goal 7. Fundamental operations with positive and negative numbers.
Goal 8. Equations and problems.
Goal 9. Elementary operations. Addition and subtraction.
Goal 10. Multiplication and division.
Goal 11. Law of exponents.
Goal 12. Parenthesis.

C. Equations.

Goal 13. Identical and conditional equations. Axioms. Dependence.
Goal 14. Problems.

Division III

A. Fundamental operations with algebraic expressions.

Goal 1. Addition and subtraction.
Goal 2. Multiplication and division.
Goal 3. Symbols of aggregation.
Goal 4. Equations.
Goal 5. Problems.

B. Special products and factoring.

Goal 6. Monomials and olynomials.
a. Multiplication and factoring.
Goal 7. Multiplication of polynomials.
Goal 8. Factoring the general trinominal.
Goal 9. Problems.

Division IV

A. Fractions.

Goal 1. Reduction of fractions.
Goal 2. Addition and subtraction.
Monomial and binomial denominators.
Goal 3. Multiplication.
Goal 4. Division.
Goal 5. Signs of terms.

B. Fractional equations.

Goal 6. Solution of general type.
Goal 7. Literal and formula.
a. Changing the subject.
Goal 8. Derivation of formulas.
Goal 9. Problems.

Division V

A. Ratio, proportion and variation.

Goal 1. Ratio and variation.
Goal 2. Proportion and variation.
Goal 3. Graphing.
Goal 4. Variation. Direct and inverse.

B. Simultaneous linear equations.

Goal 5. Graphic solution.
Goal 6. Solution by addition or subtraction.
Goal 7. Solution by substitution.
Goal 8. Fractional.
Goal 9. Problems.

Division VI

A. Powers and roots.

Goal 1. Powers.
Goal 2. Roots.
Goal 3. Surds. Addition and subtraction.
Goal 4. Surds. Multiplication and division.

B. Quadratic equations.

Goal 5. Solution by factoring.
Goal 6. Solution by completing the square.
Goal 7. Formula.
Goal 8. Simultaneous equations. A quadratic and a linear.
Goal 9. Simultaneous quadratic equations.

Division VII

A. Numerical trigonometry.

Goal 1. Functions.
Goal 2. Tables.
Goal 3. Problems.

To illustrate the material as it is given in the separate units the following is an excerpt from Division IV.

Goal 1. *Reduction of Fractions. What you learned in arithmetic about fractions.* Do you remember all the things that you can do to a fraction? Try to enumerate them. Do you remember the definition of a fraction? Perhaps you learned that a fraction is one or more of the equal parts of a number. For example, the fraction $\frac{3}{5}$ means that you have taken three of the five equal

parts into which the whole has been divided. But how do you like this definition? A fraction is an expressed division. It has been said that we got that definition in this way. When people first began to do divisions it was written $3 \div 5$. After it became familiar and folk got in such a hurry they became careless and wrote it without the bar, $3 : 5$. Then more familiarity and more speed reduced it to $3|5$. As time went on you can see that it could finally come to the form that we now use, $\frac{3}{5}$. So that is why we call it an expressed division. It means three divided by five. Sometimes it is called the quotient, too.

The fraction a/b means a divided by b, and is read "a divided by b." It is called an algebraic fraction and a and b are called the terms of the fraction. They have the old name also, a being above the line, is the numerator, and b being below is the denominator.

Mental Exercises I

In the fractions, (a) ab/cd, (b) 2a/xy, (c) m^2/n, (d) 3xy/5rs, what are the terms of each fraction? What are the numerators of the first and third fractions? What is the denominator of each fraction?

Changing the form of fractions without changing their value. You were asked to recall the different things that you could do to a fraction. What has been done to the fraction $\frac{6}{8}$ to make it equal to $\frac{3}{4}$? Would arithmetic allow you to do them? Here is another, $\frac{35}{40} = \frac{7}{8}$. What has been done this time? Is it correct? What is your authority? Now if you can do such things to arithmetical fractions, why not to algebraic fractions? Your authority for the above will hold good for any fraction.

Illustrative Exercises I

1. c/a = cd/ad. 2. ar/rx = a/x. 3. al/nl = a/n. 4. t/s = ty/sy.

Explanation. In the first and fourth fractions the terms of each fraction have been multiplied by the same number. The terms of c/a were multiplied by d and the terms of t/s by y. In the second and third the terms have been divided by the same number. The terms of ar/rx were divided by r and the terms of al/nl by l.

Mental Exercises II

What has been done to the first fraction to make it equal to the second in each of the following exercises?

1. $1/2 = 2/4$
2. $2/3 = 4/6$
3. $1/5 = 2/10$
4. $12/16 = 3/4$
5. $15/20 = 3/4$
6. $18/24 = 6/8$
7. $a/b = ar/br$
8. $xy/cd = 2xy/2cd$
9. $3x/m = 9x/3m$
10. $mx/my = x/y$
11. $rs/cs = r/c$
12. $rs/cs = r/c$?
13. $14s/35x = 2s/5x$
14. $2cd/6ac = d/3a$
15. $6abc/12bde = ac/2de$

Has it occurred to you by this time that you have been following definite rules to get the last list of exercises? Try to complete the rules by filling in the blank spaces in the following:

1. Multiplying both the numerator and denominator of a fraction by the ..... number does not change the value of the fraction.

2. Dividing both the ..... and ..... of a fraction by the same number does not change the value of the fraction.

The first goal on reduction follows:

Practice Exercises I

Fill in the missing terms in the following exercises.

1. $1/2 = ?/4$
2. $3/6 = ?/12$
3. $3/5 = 9/?$
4. $7/8 = 49/?$
5. $6/9 = 2/?$
6. $10/12 = 5/?$
7. $15/25 = ?/5$
8. $24/30 = ?/6$
9. $x/2x = ?/2$
10. $2y/10y = 1/?$
11. $3a/2a^2 = 3/?$
12. $9ab/12abc = ?/4c$
13. $2x/3 = ?/9ax$
14. $2z/3y = ?/12zy$
15. $3A/4B = 3A\dfrac{(a-b)}{?}$
16. $ax/by = ?/by\ (m-n)$
17. $x/x-y = ax/?$
18. $3a/x - 4 = ?/x^2 - 16$
19. $a/a + b = ?/a^2 - b^2$
20. $5R/x + 5 = 5R\ (x - 5)/?$
21. $a - b/a + b = ?/a^2 + 2ab + b^2$
22. $x - y/x = x^2 + xy - 2y^2$
23. $a + 1/a + 2 = a^2 + 4a + 3/?$
24. $\dfrac{a^2 b + ab^2}{a^2 + 3ab + 2b^2} = ab/?$
25. $\dfrac{a^2 - 9}{a^2 + 6a + 9} = \dfrac{a - 3}{?}$

Time Saved. Mention was made that the pupils of a class completed the course as much as eight weeks before the end of the term. In so doing, just that much time was saved for the

pupil. The time saved could be spent advantageously in a number of ways. In this specific case the course was enriched by bringing in supplementary materials in algebra, and intuitive and algebraic geometry. Also, assistance was given the instructor in checking the work done by the slower pupils and in keeping the records.

It is not intended that your inference from the above discussion shall be that it is recommended that individual instruction will cure all ailments. The experiment is to be carried further. After that is done perhaps definite conclusions can be drawn and specific recommendations made.

WHAT AMOUNT OF ALGEBRA IS RETAINED BY COLLEGE FRESHMEN?[1]

By Professor WALTER CROSBY EELLS
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The object of this paper is to report the results of giving the Hotz high school algebra scales to a group of freshman students at Whitman College, and to compare these results with those in other schools and with the Hotz standards of high school achievement.

The writer is chairman of a committee of the Inland Empire Council of Teachers of Mathematics which is supervising the giving of the Hotz tests to several thousand algebra students in about seventy-five high schools in the States of Oregon, Washington, Idaho, and Montana this year. A full report of this general investigation will be available later in the year. As one phase of it, however, the tests were given, under varying conditions, at Whitman College, at Washington State College, and at the Cheney State Normal School.

At Whitman College there are two freshman courses in mathematics, one (referred to hereafter as the "special course"), intended for students who expect to take further courses in mathematics, giving the preparation necessary for a course in calculus, and using Young and Morgan's "Elementary Mathematical Analysis" as a text; the other (referred to hereafter as the "general course"), intended for students who expect to take only a single year of college mathematics, in which a broader but less intensive study is made, using as a text Kenyon and Lovitt's "Mathematics for Students of Agriculture and General Science." Mathematics is not a required course for freshmen but was elected this year by about forty per cent of the class.

To all students in both courses the complete set of Hotz Algebra Scales, Series A, was given on the opening days of the college year in September, one test being given each day. No special effort was made to review high school algebra, but the subject of the next test was in some cases announced daily, and some students did some reviewing individually.

[1] Prepared for the Cincinnati meeting of the National Council of Teachers of Mathematics, Feb. 21, 1925.

The number of students taking the tests varied slightly from day to day, but with an average of 81, two-thirds of whom were in the special course and one-third of whom were in the general course. Fifty-one had had one and a half years or more of algebra in high school; thirty had had but a single year of it. The students with one and a half years of algebra had for the most part had their third half year in their senior year of high school, so that it was comparatively fresh in mind. Those with only one year had not studied it for at least three years, and in a few cases longer.

A general summary of results and comparison with the Hotz standards for three, six and nine months' achievement, is given in the following table of median scores:

	Number of Students.	Addition and Subtraction.	Multiplication and Division.	Equation and Formulas.	Problems.	Graphs.
Perfection		**12.0**	**12.0**	**12.0**	**10.0**	**8.0**
Hotz Standards						
9 months		7.9	7.9	7.8	5.6	5.6
6 months		6.8	6.3	7.1	4.9	3.7*
3 months		5.0	5.3	4.9	4.3	1.9*
Whitman Freshmen						
With 1½ years or more of algebra	51	8.8	7.9	9.1	7.5	7.1
With 1 year only of algebra	30	6.0	5.7	6.5	6.6	5.5

A comparison was also made between the students in the special course and in the general course. While this seemed to show a marked advantage in favor of those in the special course, a further analysis showed that this difference was almost entirely due to the fact that this group contained many more students with one and a half years of preparation. The sum of the median scores in the five tests for the 14 students in the general course who had only one year of algebra was 29.3; for the 16 in the special course with similar preparation it was 31.1—a difference of less than two problems. In two of the tests there was a difference between the two classes of only 0.1. The really significant division is on the basis of amount of preparation, as presented in the table.

* Interpolated from Hotz value for 4½ months, 2.8.

Results and Conclusions

An examination of the Hotz scales shows that they measure quite different types of algebraic ability. The first three are largely tests of manipulative ability—formal algebra. The problem scale is essentially a test of ability to think mathematically. The graph scale combines both the preceding elements to a certain degree but also requires graphic ability of a different type from either.

The number of students to whom the test was given was comparatively small; there was lack of homogeneity in the group as regards preparation, time elapsed, purpose and efficiency of instruction; and other conditions can be mentioned to indicate that the test was not as satisfactory as might be desired. But with a full realization of the limitations on the study imposed by these facts, the results seem to indicate the probable reasonableness of the following tentative conclusions:

1. Students who have had but one year of high school algebra have, upon entering college, ability in technical algebraic manipulation equivalent to that of high school students who have had from four to five months of algebra, or approximately one semester. In graphic work they have the equivalent of nine months of high school work. In problem solving ability they are somewhat in excess of the ninth months' standard.

2. Students who have had one and a half years or more of high school algebra have, upon entering college, ability in technical algebraic manipulation distinctly superior to the nine months' high school standard in all tests except that in multiplication and division, and equal it in this test. On the basis of a rather questionable extrapolation the statement may be ventured that they probably have the equivalent of eleven or twelve months' training in the formal aspects of algebra. In this respect they are in marked contrast with the students whose preparation is but one year of algebra with their equivalent of a semester of manipulative skill. In graphic work they are somewhat superior to the nine months' standard but in this respect they do not show such a marked contrast with the students who have had only one year of preparation. In problem solving ability they are much superior to the nine months' standard, but in this ability they

differ comparatively silghtly from those with only one year of algebra. There is a difference of less than one problem in the median scores of the two groups in this test which is distinctly less than the difference between the two groups in any of the other tests.

3. As regards thought power and problem solving, students with one and a half years of algebra show comparatively little improvement over those with only one year (0.9); as regards graphic ability, they show moderate improvement (1.6); as regards technical manipulation, they show marked improvement (2.8, 2.2, 2.6).

4. It is highly desirable for the prospective college student of mathematics to take the third semester of algebra in high school, principally to strengthen his ability in formal manipulation and technical ability.

5. For the average class of college freshmen, entering with one and a half years of previous study of algebra, the instructor is not justified in assuming technical algebraic ability much in excess of that secured in the high school at the end of one year's study of algebra.

6. For the average class of college freshmen, entering with one year only of previous study of algebra, the instructor is not justified in assuming technical algebraic ability much in excess of that secured at the end of one semester of high school algebra.

7. For problems involving graphic work or thinking ability and mathematical analysis standards considerably higher than those indicated in the two immediately preceding paragraphs may be assumed.

8. The conclusions stated above, insofar as they are valid at all, are valid only for the selective group (about forty per cent in this case) of freshmen who take college mathematics as an elective. The scores would all be lower and the conclusions probably greatly modified, were mathematics required of all freshmen.

9. It is highly undesirable to attempt to teach students with only one year's preparation in algebra in the same college class with those having had one and a half year's work, and who therefore possess approximately twice the technical ability.

One of the most interesting results is the marked ability in problem solving shown by both groups of freshmen as compared with the Hotz standards. This has been stated in paragraph 3 above, but is deserving of further consideration. The students with one year of algebra scored less than the six months' standards in the tests of manipulative ability, but distinctly higher than the nine months' standard in problem solving! Relatively the highest score was made in problem solving by the students with one and a half years of algebra, while the high school students according to the Hotz standards made relatively the lowest score in this same test.

If these results are really valid, they are very significant. What teacher of elementary algebra has not felt that the teaching of "reading problems," mathematical thinking as distinguished from mathematical manipulation, was not the most difficult and least successful although the most important part of her mathematical task? Is it not gratifying then to find that this ability for mathematical thinking, so difficult to inculcate, really remains and even increases long after ability in formal manipulation is largely forgotten? Part of this ability may have been acquired subsequent to the course in algebra. It is not possible to determine what part of it may be due to the training in reasoning given by the course in plane geometry, or by the courses in physics and other subjects requiring analytical reasoning, and what part to the greater maturity of the student. But whatever the cause, or combination of causes, it certainly should be a source of encouragement to the teacher of elementary algebra to know that when the high school student does reach college he then *has*, whether acquired entirely or only partially in the algebra course itself, a markedly greater ability in thinking mathematically than he has in algebraic manipulation. And of the two, the former is undoubtedly the consummation most devoutly to be desired as the *summum bonum* of algebraic study.

This conclusion may be accepted with a degree of caution, however, since it is quite at variance with that given by Professor Woody as a result of his giving the Hotz tests to the senior class "in one of the large high schools in Michigan," as reported in *School and Society* for September 9, 1922 (Vol. XVI, p. 303). He finds that high school seniors who had had one year

of algebra as freshmen made a median score of 5.6 on the equation scale, but of only 4.7 on the problem scale. Woody's general conclusion is:

"In general this investigation shows that the seniors had retained a relatively large amount of the knowledge of the more formal aspect of algebra but a comparatively small amount of the more complex problem aspect of the subject."

Latent Ability

As has been noted these tests were given at the beginning of the college year, without specific review. Would the results have been markedly different if a few days of systematic review had been given first? They measured the *status quo* on the opening days of college. Was there a reservoir of further algebraic knowledge and ability really available at the expense of a little review?

In a partial effort to throw light on this question, the addition and equation tests were repeated with the class in the general course at the beginning of the second term, the first week in January, after a term's study of freshman mathematics. No special emphasis was placed upon the subject of addition and subtraction during the term except as it came in often incidentally in the solution of other problems. But there was a marked study of the solution and interpretation of linear and quadratic equations.

For the group of students who actually took the test twice, the median score in addition and subtraction increased from 5.8 to 6.9, but still was the equivalent of only six months of elementary algebra. The incidental review did not uncover any marked reservoir of hidden knowledge! Three students actually made poorer scores on the second attempt than on the first.

For the equations test the situation, of course, was quite different. The median score increased from 7.4 to 9.5, much above the nine months' standard. But much of this was doubtless due to the specific instruction in equations given. Mathematical knowledge does not seem to "come back" as readily as is sometimes supposed!

Further light on the question is given by the results found at Washington State College and the Cheney State Normal School. In these two institutions the addition and equation scales were given under different conditions. The results in these two institutions and in Whitman for these two tests were as follows:

	Number of Students	Addition and Subtraction	Equation and Formula
Whitman—Both Classes	81	7.0	7.7
Whitman—Special Course	54	7.8	8.2
Washington State College—Engineers	98	7.3	9.1
Cheney State Normal School	94	5.6	6.7
Whitman—One Year of Algebra	30	6.0	6.5

The conditions under which the tests were given at Whitman have already been described. At Washington State College "all the students who took the tests are freshmen in engineering and have had about seven weeks in trigonometry but no algebra since their high school days." Assuming that they are much the same quality of students as are taking the special course at Whitman, the scores as far as they are comparable do not show any marked improvement as a result of the algebraic review incidental to seven weeks' work in trigonometry. At the Cheney Normal School the tests were given after ten weeks of study to the freshmen in mathematics who had had "no algebra since high school, and the majority only one year of algebra." These may therefore be roughly compared with the students at Whitman College with only one year of algebra, and they do not show any marked improvement as a result of ten weeks' additional mathematical study in the normal school.

A COMPARISON OF TWO METHODS OF ARITHMETIC PROBLEM ANALYSIS

By JOHN R. CLARK and E. LEONA VINCENT
The Lincoln School

Purpose. The purpose of this study was to compare the relative merits of two different methods of arithmetic problem analysis. The two methods were the conventional method and what, for lack of a better name, we have called the "graphical" method.

Definition of Methods. The former method directs the pupil to analyze the problem by ascertaining

1. What is asked for in the problem.
2. What facts are given in the problem.
3. How should these facts be used to secure the answer.
4. What is the answer to the problem.

The "graphical" method directs the pupil to determine what is to be found in the problem, what it depends upon, what each of these dependents in turn depends upon, and so on until he has unravelled the essential facts and relationships in the problem. A problem solved by each of these methods will help in understanding their nature.

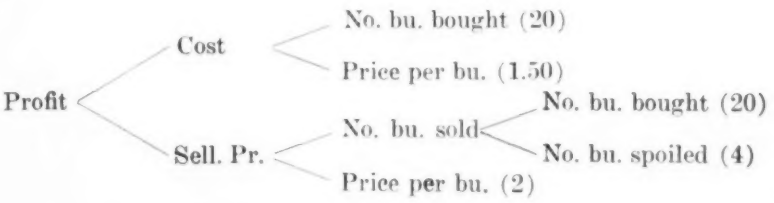
Illustrative Problem

A grocer bought 24 bu. of potatoes at $1.50 per bushel. Four bushels spoiled. The others were sold at $2.00 per bushel. Find his profit.

Solution by Conventional Analysis

1. **What is asked for in the problem?** ***Grocer's profit.***
2. **What facts are given in the problem?** ***Number bushels bought (24); number spoiled (4); cost per bushel ($1.50); selling price per bushel ($2.00).***
3. **How should these facts be used to secure the answer?** ***(a) 24 × ($1.50); (b) 24 − 4; (c) 20 × $2.00; (d) c − a.***
4. **What is the answer to the problem?** ***$4.00.***

Solution by the "Graphical Analysis": In this method the analysis is accompanied by a picture of the relationships or dependences between the elements of the problem, as suggested here.



The pupil is directed to think of the diagram as illustrating the following: To find the profit, I would have to know the *cost* and the *selling price;* to find the cost I would have to know the *number of bushels bought* and the *price per bushel;* to find the selling price I would have to know the *number of bushels sold* and the *price per bushel.*

The Subjects. The pupils of seventh and eighth grades of the Lincoln School were used as subjects. There were some 80 children in all who were divided into two equivalent groups on the basis of arithmetic reasoning ability and intelligence. The reasoning ability was measured by the Stone Reasoning Test I and the Arithmetic Reasoning Section of the Stanford Achievement Test, Form A. From a study of the variability of the scores on each of these tests equal weight was given to them in a combined score by multiplying the Stone scores by three. The intelligence was measured by the Stanford Binet Test. Due to the fact that the relationship between mental age and reasoning scores showed a correlation (Pearson) of only +.72 considerable care was given in forming the equivalent groups to matching pupils in terms of arithmetical reasoning scores, mental ages, and intelligence quotients.

Experimental Materials. The material used as a means of comparing the two methods of problem analysis comprised a list of 50 problems chosen by the writers to represent as nearly as possible a random selection of text-book problems. Of these, 27 dealt with some aspect of buying and selling; 10 involved the use of proportion; 9 were based on wages, earnings, and savings; 2 involved insurance, and 2 uniform motion. These

problems were mimeographed in six sets of eight, the remaining two being placed on a separate sheet for use as demonstration problems.

The pupils using the conventional analysis were supplied with mimeographed forms on which the four standard questions appeared with appropriate blanks for their answers. The pupils using the "graphical analysis" were provided with blank sheets 6½ inches by 7½ inches, one for each problem.

On the first day of the experiment each group was introduced to the method it was to practice, the two demonstration problems being used for purposes of illustration. Since there are several ways in which any problem may be attacked the entire first period was devoted in both groups to the development and consideration of the various possibilities within the method each was to use.

During the next six recitation periods each group devoted 40 minutes to the solution of eight problems by the assigned method. The papers were then collected and the remaining ten minutes devoted to a discussion of difficulties. In this way it was possible to get from each pupil a measure of his success with each problem without help, and at the same time to give instruction in the use of the method being practiced.

The problems were then scored by giving a maximum of two points for a correct response to what was asked for, two for the correct and relevant facts, four for evidence of correct reasoning, and two for a correct numerical answer, making the total possible score on each problem for each child of ten points.

Results. Since an inspection of the original data showed that the conventional analysis was markedly superior to the "Graphical Analysis" in the beginning but at quite as great a disadvantage in the end, it became evident that a truer measure of the effect of the methods would be obtained by dividing the data into quarters than by comparing mean or median scores of the groups as a whole. The resulting data are presented in Table I.

TABLE I

	Correct Reasoning		Correct Solutions	
	Conventional Analysis	Graphical Analysis	Conventional Analysis	Graphical Analysis
First 12 problems	267	234	223	185
Second 12 problems	164	174	142	138
Third 12 problems	326	356	259	295
Fourth 12 problems	202	263	175	211

Table 1 shows that the number of perfect reasoning scores is greater in the first quarter for the Conventional Method, but that for each successive quarter it is greater and increasingly greater for the "Graphical Analysis" method. If the two methods are compared on the basis of the number of correct solutions the advantage is again decidedly in favor of the Conventional method in the first set of 12 problems, slightly in favor of it in the second set of 12, but markedly in favor of the "Graphical Method" in the last half of the list. It appears not unlikely that a longer period of practice would have shown even greater superiority for the "Graphical Analysis" method.

It became apparent in scoring the daily practice papers that occasional children were finding the method a hindrance rather than a help to the solution of the problem. In some of the examples for which correct answers were obtained the analyses were either omitted or recorded incorrectly. To compare the two methods with respect to this type of difficulty note was taken of all such cases. In 77 cases the solutions were hindered because of inability to record the analysis in the required "Graphical" form, in 187 cases for the Conventional form. This is to be expected since the Conventional form requires somewhat more of English composition for its expression.

It seems obvious too that some children do solve some problems without being able to formulate the process by which they arrive at the answer. In such cases any technique of analysis probably retards progress.

The Relative Difficulty of the Four Steps in Problem Analysis. It is important to teachers of arithmetic to appreciate the relative difficulty of the various steps in the solution of problems. A study of the data from this investigation showed only 50 errors out of a total of 2,778 attempts in selecting what was wanted in the problem. The pupils had practically no difficulty in determining what they were asked to do. There were 92 failures to comprehend or indicate the salient facts given in the statement of these problems. There were, however, 674 failures to recognize and indicate the relationships and operations essential to the correct solution. There were 402 cases in which computation only was in error. It appears, therefore, that of these four steps in problem solving the one involving the choice and order

of the operations necessary to the solution was the most difficult for our pupils. It would seem wise to give further training in this phase of problem-solving as well as in the mechanics of computation.

While the primary purpose of the experiment was to determine the relative merits of the two methods with respect to the soluion of the list of problems described above it seemed advisable to give at the end of the practice period duplicate forms of the reasoning tests that had been given at the outset. Accordingly the Stone Reasoning Test II (with adjustment for differences of scoring recommended by Stone) and the Arithmetic Reasoning Section of Form B of the Stanford Achievement Test were given and weighted as in the initial test, Stone 3 and Stanford 1. The children were told that they need not use the particular methods they had been practicing but to solve these problems by any method they chose.

The pupils in the Conventional Analysis group made an average gain (algebraic) in Stone scores of 1.0, in Stanford scores of 5.8, and in weighted reasoning scores of 8.8. The "Graphical Analysis" group gained .74 in Stone scores, 3.8 in Stanford scores and 6.02 in weighted reasoning scores. This seems to be in contradiction to the tendencies noted in the data obtained from the daily practice scores. Since the "Graphical Analysis" method was showing more accurate reasoning at the close of the practice period one would expect this group to register higher scores on the final tests. It will be recalled that the pupils were not required to use any formal written analysis in the final tests. Since no evidence was given that the children were using either of the formal methods practiced before we assume that they fell back to the old, informal methods of problem solving, which were, because of past teaching, more like the Conventional than the "Graphical" method. This would naturally tend to favor the final scores of the Conventional group.

From experience in the teaching of mathematics we know, too, that any partially formed skills or procedures impede rather than facilitate the operation of old habits in the same field. It is obvious that our pupils did not practice the "Graphical" method long enough to have it replace naturally their former

methods of attack. When released from the necessity of using the unfamiliar procedure they reverted to the familiar though now somewhat impeded methods. The writers consider, therefore, that the data of this investigation do not present any real conflict.

Relationship Between Amount and Quality of Practice, Gains in Reasoning Scores, and Intelligence. A study of the individual gains and losses from the initial to the final tests showed some unexpected results in both practice groups. There were very large losses as well as large gains in combined reasoning scores, the changes in scores in the Conventional group varying from — 28 to + 41, and in the "Graphical" group from — 43 to + 33. When the largest gains and losses were studied in relation to the total individual daily practice scores there appeared to be no relationship whatever between amount and quality of practice and the resulting gain or loss. Table II shows the daily practice scores of the two greatest gains and the two greatest losses for each practice group.

TABLE II

SHOWING SOME OF THE GAINS AND THEIR ACCOMPANYING DAILY PRACTICE SCORES

"Graphical" Group			"Conventional" Group		
Pupil	Gain	Practice Scores	Pupil	Gain	Practice Scores
J. E.	— 43	414	S. S.	— 28	322
J. K.	— 21	260	A. W.	— 19	382
J. B.	+ 33	414	J. H.	+ 41	231
A. M.	+ 28	314	R. B.	+ 41	297

The relationships between the gains, daily practice scores, and intelligence are shown in Tables III-a and III-b.

TABLE III-a

SHOWING RELATION BETWEEN INTELLIGENCE QUOTIENTS, DAILY PRACTICE SCORES, AND GAINS

Intelligence Quotient	Number Cases	Practice Scores	Average Gains
140-149	10	369.2	+ .7
130-139	11	343.5	+ 2.4
120-129	17	314.2	+ 8.9
110-119	14	325.6	+ 4.9
100-109	11	272.8	+ 9.5
90- 99	6	248.1	+11.0

TABLE III-b

SHOWING RELATION BETWEEN MENTAL AGES, DAILY PRACTICE SCORES, AND GAINS

Mental Age Dec. 1, 1924	Number Cases	Practice Scores	Average Gains
19-0 to 19-11	5	388.4	— 5.0
18-0 to 18-11	9	369.2	+10.9
17-0 to 17-11	16	321.8	+ 0.8
16-0 to 16-11	17	310.9	+ 5.8
15-0 to 15-11	10	301.4	+13.0
14-0 to 14-11	4	277.5	+ 4.0
13-0 to 13-11	6	236.5	+ 9.2

When Table III-a is studied it can be seen that there is a marked positive relationship between intelligence quotient and daily practice score, but an equally marked negative relationship between gains and intelligence quotients. The pupils with the highest intelligence quotients made the highest practice scores but the smallest gains.

Two explanations may be offered for this. It is evident that when a group is near the limit of its ability to improve the gains from practice will be less than will those of a group which has more room for improvement.

Professor Thorndike has offered the suggestion that the brighter children suffer in growth if forced into too rigid technique, whereas duller children need such technique to bring about their best development. It is probable that both factors operated to make the relationships of Table III what they are.

Table III-b shows that there is an even more marked relationship between mental ages and practice scores than between those scores and intelligence quotients. The gains, on the other hand, seem to have no relation to mental age unless one can see in the figures a slight tendency for the higher mental ages to have profited least from the work of the experiment.

Conclusions

While the number of cases included in this study was small and the time too brief to justify any positive statements, the writers suggest the following tentative findings.

1. In this study the pupils who practiced the "Graphical Analysis Method" improved during practice more than the group

who practiced the Conventional Analysis Method, and gave promise of still greater improvement had the period of practice been longer.

2. Seventh and eighth grade pupils seem to have little difficulty in recognizing what is wanted in the problem and in selecting the relevant data. They do have, however, considerable difficulty in choosing the operations and organizing the relationships involved in the solution of the problem. In administering remedial treatment to children who have special difficulty with problem solving it would seem wise to give special attention to the setting up of associations between the cues, situations, and contexts appearing in "described" problems and the nature and order of fundamental operations which make possible their solution.

3. There was a marked positive relationship between intelligence quotients and daily practice scores, but an equally marked negative relationship between intelligence quotients and gains in reasoning as measured by an initial and a final test with the Stone Reasoning and Stanford Achievement Tests. There seemed to be no definite relationship between gains in reasoning scores and mental ages, although the positive relationship between daily practice scores and mental ages was even more marked than between daily practice scores and intelligence quotients. It is quite possible that although duller children profit by a definite technique brighter children are hampered by it.

4. There are many problems the formal analyses of which are almost impossible although their solutions are easy. It is important for teachers to know in advance those types of problems. Formal analysis should be considered a tool or ally to be used only when the solution is not otherwise possible.

TEN REASONS WHY PUPILS FAIL IN MATHEMATICS[1]

By DR. W. J. OSBURN
State Department of Education, Madison, Wis.

Why does William stare at you in helpless confusion when you ask him a question? Why is Mary failing in mathematics? Why are mathematics classes so often beset with hopeless groups of tail enders? These are burning questions for teacher, parent and pupil. In days gone by puzzled physicians when pressed for a reason as to why their patients did not recover hit upon the happy idea of saying that the patient was possessed of some evil spirit and backed their assertions by all kinds of resorts to conjuring and necromancy. Today puzzled teachers when pressed for a reason why their pupils do not learn are using the same strategy. They tell us that their pupils are possessed with a spirit of carelessness, indifference, laziness or lack of perseverance. Long before the final chapter is written concerning the teaching of mathematics these reasons wil doubtess prove to be as great subterfuges as were those of the ancient medicine men.

Neither diseases nor failures in mathematics are caused by evil spirits. It is the purpose of this paper to set forth ten reasons why pupils fail which are reasonable and capable of proof by experimentation. The root of the problem lies in the existence of individual differences in pupils to an extent hitherto undreamed of. The trouble is much increased by the fact that this heterogeneous horde of humanity insists upon coming to high school and bringing its differences along with it.

The chief difference which needs concern us here is that which exists in the ability to infer conclusions. Pupils who are good at inference—who can apply in novel situations what they have previously learned—such pupils will have no trouble in mathematics. If learning were a stairway these pupils could mount at the rate of two or even three steps at a time. But the pupil who has little or no ability to infer cannot climb the stairway even one step at a time. If he ever gets mathematics someone must build in more steps and smaller ones. Little or no cognizance has been taken of this fact by those who have written our textbooks. The only hope of relief now is through the

[1] Read before the National Council of Teachers of Mathematics, Cincinnati, Feb. 21, 1925.

preparation of the needed practice material by the teacher. It is impossible within the limits of this paper to present any material of this sort. All that can be done is to indicate the following ten places in the teaching of algebra where such supplementaton is badly needed.

1. Exercises in vocabulary.
2. Exercises to develop the meaning of symbols.
3. Training in silent reading to eliminate failure to see and use all of the data.
4. Training in the use of inverse relations.
5. Training to prevent harmful transfer.
6. Drill in horizontal addition and subtraction.
7. Training in reading between the lines.
8. Help for the pupil when conditions seem contradictory.
9. Training pupils to generalize.
10. Proportion.

The following illustrations, all but two, are taken from the first ten pages of a widely used algebra text.

1. Problems for which exercises in vocabulary are a necessary prerequisite.

(a) The perimeter of a certain rectangle is 256 ft. It is three times as long as it is wide. Find its dimensions.

(b) In how many years will $150 treble itself at 5% simple interest?

The troublesome words are rectangle, perimeter, dimensions and treble.

The child who is capable of inferring what to do in new situations will either know or know how and where to find the meanings of these words. The child who is poor at inference will be unable to proceed and worse still, he will be unconscious of the cause of his trouble.

2. Problems for which training in the meaning of symbols is a necessary prerequisite.

If m represents one month and $y = 12m$, express $y + 5m$ in terms of m.

The fact that m can represent one month is easy for those who can infer and mystifying to those who cannot. To them m is m and that is all there is to it.

> "The primrose by the river's brim
> A yellow primrose was to him
> That and nothing more."

The situation is further complicated by the word "express" and the phrase "in terms of." To the pupil who cannot infer,

express means the American Express Company and terms has something to do with the length of school. He neither knows nor sees the meanings as used here.

3. Problems in which some of the data are likely to be overlooked.
The sum of two numbers is 160. The greater is four times the lesser. Find each.

The typical reaction of the pupil who cannot infer to this problem is

$$\begin{aligned} \text{Let } x &= \text{the less} \\ \text{Then } 4x &= 160 \\ x &= 40 \end{aligned}$$

This is all he sees to the thing though there may be a dim feeling that something is lacking. He has no clue to guide him into the unknown territory and he has learned through bitter experience that venturing into unknown intellectual territory without a clue usually results in ridicule from his classmates. Hence he remains silent.

4. Problems in which training in the use of inverse relations is a necessary prerequisite.
(a) At what per cent simple interest will $825 gain $165 in four years?
(b) See problem b under 1.

Most ninth grade children know that you multiply the principal by the rate by the time to get the interest. But now they are asked to find the rate or the time. Those who can infer will know what to do. Those who cannot will see a problem totally unlike anything which they have ever seen before.

5. Exercises in which practice is needed to prohibit harmful transfer.
(a) In the term $3a^2b$ indicate which is coefficient, which is factor and which is exponent.
(b) Simplify $(6 - 3)(17 - 2.5)$.
(c) Simplify $23 - 2.6 - 4 \div 2 + 6$.

Correct inference implies the ability to see small differences and likenesses. Pupils who cannot infer are incapable of perceiving these differences. The coefficient is usually a number written in front of a factor. An exponent is much like a coefficient placed just above and to the right of a factor. Furthermore, it is just in front of the factor which follows. The period meaning multiplication, is very little different from a decimal point. The presence or absence of a parenthesis seems of little consequence. Yet all of these are significant factors in the ability to respond correctly and the pupil who overlooks them is obviously headed toward trouble.

6. Exercises in which drill in horizontal addition and subtraction are necessary.

In collecting terms in an equation mistakes in addition and subtraction are notoriously common. This is because the pupil's previous experience has been almost entirely restricted to column addition. To make the transfer to the horizontal type is impossible for some pupils unless much practice is afforded.

7. Problems in which reading between the lines is a prerequisite. Express $4h + 3m$ in seconds if h and m stand for the number of seconds in an hour and in a minute respectively.

In problems of this sort the pupil must select the right element from his previous stock of knowledge and must know just where and how to use it. The missing elements here, of course, are 60 sec. = one minute and 60 min. = one hour. Failure at this point is very common and very disastrous in problem solving.

8. A situation in which help is necessary because the condition seems contradictory.

The worst case of this sort in the whole field of secondary mathematics is the pupil's first encounter with negative numbers. To some pupils there will come a feeling that all that they have considered as fundamental is being uprooted and overthrown. In arithmetic they have been told that they cannot subtract a larger number from a smaller one and that they *must not* subtract a small digit from a larger one just below it and now they are taught to *do* all of this. The placing of the minus sign in front of the result means little to them. No one understood negative numbers until more than a century after America was discovered. It is small wonder that the ninth grade finds them puzzling.

9. Situations in which practice is needed in generalization. This is extremely difficult for the slow pupil but there is at least a little hope.

Problem 1b is a specific case. It would be good if all of the pupils would notice that the answer is not dependent upon the $150 or on any specific amount. It would be good also if all who can solve equations in x and y could solve the same equation in v and t or i and k. But every teacher of mathematics knows that this is one of the most difficult objectives to attain.

10. Problems in which practice in proportion is prerequesite.
A flagstaff casts a shadow 12 yards long; at the same time a man 5 ft. 10 in. tall casts a shadow 35 inches long. How high is the pole?

To state proportions one must be able to see the analogy which is involved. The analogy in turn is based upon a similarity. The question in this case is *how* is a man like a flagstaff. Any person who has used a Binet test knows that this is difficult as a general proposition. It is still more difficult when some particular resemblance is wanted. In this case the only resemblance that counts is the fact that each casts a shadow. But this is not all. The child must recall that upright objects cast shadows which are directly proportional to their length. He is then ready to say that the height of the man is to the length of his shadow as the height of the flagstaff is to the length of its shadow. After this the ability to read and substitute the proper values is all that is needed to state the proportion correctly. Every proportion problem involves all of these steps. Hence a child who is able to state a proportion correctly is one who can take four steps in one. Such ability is not to be expected in the case of the slow pupil. He will have to have practice on each of the four levels.

Summary

1. A pupil fails for definite reasons. Teachers should not try to explain his failure in terms of some vague spirit of which they are said to be possessed.

2. Ten of the more important causes of trouble have been named and illustrated.

3. Teachers who are afflicted with slow pupils will have to scrutinize the exercises and problems very carefully to see that no child is expected to make more steps at one time than he is capable of making. This is often difficult to do because many teachers do not realize the existence of obstacles which are insurmountable to the slow pupil.

4. After the difficulties are identified it is usually possible to devise types of drill which will take care of them. Up to date no such drill exercises have been published. That job must be taken care of in the immediate future if courses in mathematics are to be of value to the slow pupil.

5. All of the illustrations in this paper have been drawn from algebra. The difficulties themselves, however, are present for the most part in every type of mathematical study.

USES OF CARDBOARD

By MISS RUTH LANE
University School, Iowa City, Iowa

All mathematics teachers who read this article have no doubt found various uses for cardboard, probably some of them the same as those I describe, for only part are original with me.

In the ninth grade we use large charts of manila cardboard 25 by 30 inches upon which the problems for board work, and sometimes seat work, are printed. The chart is put in a place where all can see. There is no waste of time in reading problems. A student works at his own speed and as soon as he finishes he can take his seat and begin studying the next day's assignment. When desired part of the class can have one or two problems which are more difficult than those of the rest. For making these charts we use two printing sets, a large size and a smaller size for exponents. The charts are numbered and filed. A list of the numbers, the type of work on each chart, and the answer cards are filed for future use. Several departments use these sets jointly.

Sometimes we prefer to have different problems given to each student to correct his particular trouble. If it is algebra, we select some suitable cards from the set by Briggs or at times choose problems from other sources and put them on cards of our own making. The cards containing problems of one type are then fastened together with a rubber band and can be used again. We use this plan in third semester and advanced algebra also.

For geometric drawings the practical use of cardboard seems almost unlimited. If all the quadrilaterals are drawn on a large sheet with their names below, and the sheet posted above the black board in the front of the room, the teaching of the quadrilaterals and the spelling of the words is about half done. Before formal proofs are begun, considerable construction work is done. All three altitudes are constructed in a triangle. Similarly, the bisectors of the angles, the perpendicular bisectors of the sides, and the medians are drawn. The students draw a triangle on cardboard, construct the medians, cut out the triangles, and in

class the next day show whether or not they have been very accurate in finding the center of gravity or centroid by balancing their triangles on a pencil point. Then for review and comparison each student draws a large triangle on a sheet of paper or cardboard, and constructs on it each of the four sets of lines, using different colors of ink and putting the key in the corner of the sheet. A boy in the class suggested he would like to make one large enough so that the lines could be seen anywhere in the room and we could use it for reference later. It is just about as convenient to have some of these charts about a mathematics room as to have a map in a history room. For this a gray background with bright colors for the lines does not show soil easily and is attractive.

When we begin studying congruency of triangles, light weight cardboard or heavy paper and scissors (borrowed from the primary department) are passed out. The class are told to make an angle of a certain size and extend each side to a certain length, then told to complete the triangle, and then to cut it out. Each fits his on several of his neighbors' triangles.

A short time before Christmas, after the number of degrees in polygons has been studied, the class is asked to make a five-pointed star, cutting it out of cardboard. Sometimes a student makes his the size some one at home wants for a part in a Christmas program or for a tree.

About a week before Valentine Day, geometric valentines are announced. To be sure it is only an assignment for one day—we could not afford to spend longer—but we talk it up and students think up original ideas and often spend considerable time at home making one. Red and other colored cardboard of light weight is available to choose from. A student's valentine may be funny, serious, or really a wall ornament, but it must be made entirely with compass and straight edge. Several of the designs have been borrowed by the art teacher for models. One of the most unique funnies was an owl mounted upon a heart. He was entirely made up of circles and a few straight lines. The more you looked at him the more you marvelled.

In solid geometry the first theorem studied, "If a line is perpendicular to each of two other straight lines at their point of intersection, it is perpendicular to the plane of the two lines," is studied from models made by all with cardboard, string, and

pencil. Sometimes we make models for one or two others. For a couple of weeks when the class is learning something of perspective drawing all draw each figure, most of them under supervision. After that the drawings are assigned to be put on large sheets of cardboard 15 by 25 inches. The students take turns making the drawings. They are assigned ahead. These are used to give the proofs from in class. There is no using of poor sketches on the board. Moreover, they are ready for reference and review at any time. The student draws fewer figures possibly than by the ordinary method, but he draws ones he is usually rather proud of. I might also mention the making of regularly polyhedrons.

Lastly, but best of all, is the use of flash cards. We are just beginning to realize how much value they can be to us. In the grades they are used in reading. In learning foreign languages they use them. Why not use them in mathematics? A couple of minutes which might otherwise be wasted by all or part of the class is sufficient time to use a bunch of cards illustrating angles formed by chords, tangents, radii and secants in various positions. The teacher announces what the set is but not each card. The student describes or names each. The answers are on the back of the cards. In another set similarly drawn the degrees in the arcs cut off are printed on them and the class tell the sizes of the angles. Still another set illustrates common tangents. Other sets are for factoring. The printing sets are used in making these cards. This is a time saving method for drill work which I believe can profitably be used for many topics.

MINUTES OF MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS HELD AT CINCINNATI, FEBRUARY 21, 1925

President Schorling called together a group of members of the Council in the ball room of the Sinton Hotel at 9:30 for informal discussion of the Council's affairs. Ways and means of strengthening local groups, increasing the influence of the Council, and enlarging the subscription list of MATHEMATICS TEACHER were the topics of greatest interest. In the course of the discussion, Professor Slaught laid before the group the experience of the Mathematical Association. Miss Gugle, Miss Worden, and Messrs. Austin, Schorling, Schreiber, Reeve, and Overman and others took part in the discussion.

On motion of Miss Gugle, the chair appointed a committee consisting of Miss Gugle, Miss Kohnky, Miss Daly, and Messrs. Overman, Slaught, Taylor, McNally, E. R. Smith, and John Stone to consider these matters and report back to the Council.

On motion of Mr. Winter, the same committee was directed to act as a nominating committee.

The chair appointed an auditing committee consisting of: Mr. R. M. Sprague, J. R. Overman and Miss Kohnky.

The program arranged for the morning was then taken up.

Professor W. W. Hart was unable to be present, and telegraphed his regret. His paper was not presented.

Miss Winona Perry of the Lincoln School read a paper on "The Possibility of Conceptualizing the Processes of Thinking in Plane Geometry."

Mr. C. N. Stokes, New Trier Township High School, Illinois, read a paper on "Individual Instruction in Ninth Grade Algebra."

Professor W. C. Eells of Whitman College, Walla Walla, Washington, sent in his paper on "What Algebra Is Retained by

College Freshmen?" This paper was not read, due to the lateness of the hour when it was reached on the program.

The afternoon session convened at 2:45 p. m.

Professor F. B. Knight, University of Iowa, read the paper written by Professor O. L. Lutes, on "The Problem of Drill in the Seventh and Eighth Grades."

Dr. J. W. Osburn, Director of Educational Measurements, Department of Public Instruction of Wisconsin, read a paper on "The Psychological Approach to Curriculum Construction in High School Mathematics."

John R. Clark read the paper prepared by Professor Upton, Teachers College, on "The Relation of Standard Tests to the Reorganization of Mathematics."

This paper was discussed by Messrs. Reeve, Osburn, Woody, Slaught, Trabue, Otis, and Miss Worden.

Professor Overman presented the report of the committee appointed at the morning session. After discussion and amendment the report was adopted. The report follows:

"Plan for Closer Affiiliation of Mathematical Associations and Club with the National Council.

The committee recommends:

"That all organizations (associations, clubs, etc.) of teachers of mathematics in the United States concerned wholly or in part with secondary or elementary school work, shall be invited either to become sections of, or to affiliate themselves with, the National Council of Teachers of Mathematics;

"That the conditions of affiliation be, first, approval by the Executive Committee as bona fide organizations, second, the filing of intention of affiliation, a list of officers, and the list of members. Associations will also be expected to furnish minutes of meetings;

"That an intensive campaign shall be undertaken for the purpose of making as large a proportion as possible of the members of these organizations subscribers to the MATHEMATICS TEACHER. To stimulate the organization of local groups, the subscription

price of the TEACHER should be reduced. Any affiliated club or section of teachers sending in 25 or more subscriptions *at one time*, should pay on $1.75 a year for the TEACHER.

"That these organizations also be urged to send representatives to the annual meeting of the Council;

"That there shall also be individual members of the Council who shall pay annual dues of $2.00, which will include subscription to the MATHEMATICS TEACHER;

"That the Executive Committee, acting as a nominating committee, shall nominate two candidates for each office to be filled and that these nominations and ballots shall be published in the MATHEMATICS TEACHER in December and a vote shall be taken by mail. The election shall be closed on February 1st and the ballots shall be counted by the Executive Committee at the annual meeting. Those receiving a majority of the votes shall be declared elected. Only subscribers to the MATHEMATICS TEACHER shall be entitled to vote;

"That the constitution be amended as necessary to accord with this report."

The nominating committee, consisting of J. R. Overman, E. H. Taylor and Miss Kohnky, presented the following nominations: President, Raleigh Schorling, University High School, University of Michigan; Vice President, Miss Winnie Daley, New Orleans; Executive Committee, Harry English, Director of Mathematics, Washington, D. C., and Harry Barber, English High School, Boston, Mass.

On motion the report of the nominating committee was adopted.

The Treasurer reported that at the present time the Council's records show a deficit which is being wiped out by the continuous payments made by the Editor and Manager of MATHEMATICS TEACHER as subscriptions are paid in, under the agreement whereby one-eighth of each subscription is credited to the Council.

The auditing committee reported that it had examined the Treasurer's books and accounts, and had found them to be correct.

The report was approved.

Mr. C. M. Austin moved the appointment of a committee to consider the matter of publishing a year book of the proceedings of the Council, and reports of investigations in the teaching of secondary mathematics, at the next meeting. After discussion the motion was carried, and Mr. Austin was appointed chairman of the committee, with authority to add two other members.

The evening session convened after dinner, with Professor Slaught in charge. The following papers were presented:

"The Content and Method of the New Curriculum in Arithmetic," by Dr. G. M. Wilson, Boston University.

"Factors of Success in Ninth Grade Algebra," by Edwin L. Schreiber, Head of Mathematics Department, Proviso Township High School, Maywood, Ill.

"The Psychological Analysis of a Section of Algebra," by Professor F. B. Knight, the State University of Iowa.

After expressing appreciation of the effective arrangements made for the meeting of the Council by the local Cincinnati group, the meeting was adjourned.

J. A. FOBERG, *Secretary.*

NEWS NOTES

The Middle States and Maryland Association of Teachers of Mathematics will hold its annual spring meeting on May 9, at Teachers College, New York City. At the morning session (10 A. M.) there will be a discussion of *General Mathematics*. Professor David Eugene Smith will speak on General Mathematics in the Junior High School; John A. Swenson will read a paper on General Mathematics in the Senior High School, and Professor R. W. Burgess, of Brown University, will discuss General Mathematics in the Junior College. At the afternoon session Professor Tomlinson Fort, of Hunter College, will conduct a conference on *A One Year Course in Plane and Solid Geometry*. He will direct attention to such questions as: If pupils are to spend only one year on demonstrative geometry, should that year be devoted entirely to plane geometry? If we decide to include materials from both plane and solid geometry, how shall these be arranged?

THE Editor of THE MATHEMATICS TEACHER wishes to secure copies of the October, 1923, and October, 1924, numbers of the TEACHER. Any person who is willing to supply one of these copies may have the expiration date of his subscription advanced two numbers. Subscribers who are willing to make this exchange should inform the Editor.

PROFESSOR W. S. GUILER of Miami University, Oxford, Ohio, has recently published a monograph entitled "Teaching Arithmetic Through Games and Other Pupil Activities." In one part of the monograph the author lists the objectives in arithmetic for each of the grades. He discovered these by examining a large number of courses of study. In the second part there is a description of the pupil activities by which the objectives enumerated may be realized. These pupil activities are for the most part games and other drill devices. They are classified by grades.

MISS MARIE GUGLE, Assistant Superintendent of Schools in Columbus, Ohio, has just revised her *Modern Junior Mathe-*

matics which she published with the Gregg Publishing Company. In revising this well-known series of books, Miss Gugle has incorporated the results of the experiences of many teachers who used the earlier edition.

Miss Mona Del Taylor of the Scott High School, Toledo, has just published with Lyon and Carnahan, *Practice Exercises in First Year Algebra.*

Mr. Wm Betz of the East High School, Rochester, New York, gave an illustrated talk on "Intuitive Geometry" at the January meeting of the New York Section of the Association of Teachers of Mathematics in Middle States and Maryland. Professor Lao G. Simons of Hunter College, New York, is President of the Association and Miss Edna Kramer of the Wadleigh High School, New York City, is Secretary.

Professor T. Percy Nunn, Director of the London Day Training College, University of London, will give a course on the Teaching and Supervision of Mathematics at the 1925 Summer Session of Teachers College, Columbia University.

The January, 1925, number of the *American Mathematical Monthly* is for the most part devoted to the consideration of the life of Benjamin Pierce. Professor Pierce was most influential in stimulating research in mathematics in America.

One of the most complete and scholarly treatises of mathematical statistics in English was published recently under the editorial supervision of Professor H. L. Reitz of the University of Iowa. The volume is called *A Handbook of Mathematical Statistics.* It is published by Houghton Mifflin Company. The contributors are the members of the Committee on the Mathematical Analysis of Statistics of the Division of Physical Sciences of the National Research Council. They include Doctors H. C. Carver, A. R. Crathorne, W. L. Crum, James W. Glover, E. V. Huntington, Truman L. Kelley, Warren M. Persons and Allyn A. Young. This book is of such advanced nature that it will be of interest to high school teachers only as a reference book.

Mr. Robert R. Goff, Director of Mathematics in New Britain, Conn., has just published with the Palmer Company of Boston, *The Second Book in Algebra.* Mr. Goff is well known for these drill books both in algebra and geometry. The growing movement to individualize instruction promises to make books of this nature more desirable as sources of supplementary material for the more capable pupils. While *The Second Book in Algebra* contains little explanatory material, it might well be used as a basic text in the needs of capable pupils.

Professor E. R. Breslich of the University of Chicago has just published the second part of the first book of his series of *Junior High School Mathematics.* The series is being published by the Macmillan Company. The full review of this series is being withheld until both the seventh and eighth grade books appear.

Professor Harold Rugg is just publishing the *Primer of Graphics and Statistics* with Houghton Mifflin Company, in which he presents the types of statistical methods that class room teachers need to use.

Miss Edith E. Morin, 4815 Windsor Avenue, Philadelphia, has the unique distinction of having secured more than fifty memberships for the National Council of Teachers of Mathematics. Mr. Harold B. Garland of the High School of Commerce, Boston, holds second place in the number of memberships secured.

Mrs. M. W. Arleigh of South Pasadena, California, is the author of a set of exercises in fractions which are accompanied by certain illustrative materials by which it is believed that the pupil can teach himself the skills in fractions.

In his last annual report, President Nicholas Murry Butler of Columbia University states: "The present-day mocking appeal to an infant that he give expression to himself represents the abdication of education. . . .

"The ability to read has well nigh disappeared if the reading be serious, instructive, or enobling; the ability to write, so

far as it exists at all, delights to manifest itself in forms of exceptional crudeness and vulgarity; the ability to perform the simplest mathematical operation is, to all intents and purposes, confined to teachers of mathematics or to specialists in that subject. Algebra and geometry, whether plane or solid, are as unfamiliar as the Laws of Manu. . . .

"The extensive and intensive study of natural science, now carried on over more than a full generation, has made no impression whatever upon the public mind. That mind continues to come to its conclusions and to formulate its choices with serene unconcern as to whether any such thing as scientific method exists."—A. D.

THE mid-winter meeting of the Association of Teachers of Mathematics in New England was held March 7, 1925, at Springfield, Mass.

Professor Ruth G. Wood of Smith College discussed "Some Changes in Freshman Mathematics Courses."

Mr. H. C. Barber of the English High School, Boston, read a paper on "Real Improvement in Algebra Teaching."

Professor V. H. Wells of Williams College discussed "The New Method of Content Examining in Mathematics."

Mr. Robert R. Goff of the High School, New Britain, Conn., presented a paper on "Simplified Calculus for High Schools."

Professor William L. Machmer of Amherst Agricultural College read a paper on "The Mathematics of an Agricultural College."

The Council for 1925 for the Association of Teachers of Mathematics in New England is as follows: Professor Lennie P. Copeland of Wellesley College, President; F. E. Lane of Milton Academy, Vice President; Harry D. Gaylord of Browne & Nichols School, Cambridge, Secretary; Harold B. Garland of the High School of Commerce, Boston, Treasurer.

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Continued from the March number.

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